Introducing “Feedback” Among the Origin-Destination, Mode and Route Choice Steps of the Urban Travel Forecasting Procedure in the EMME/2 System

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Introduction

The issue of “feedback” in the traditional four-step urban travel forecasting procedure (UTFP) has re-emerged recently with the impetus of the Clean Air Act Amendments of 1990 and the Intermodal Surface Transportation Efficiency Act of 1991. The Federal Highway Administration now requires that Metropolitan Planning Organizations implement “feedback” in the UTFP. A sound and appealing solution to this problem is a model that combines the trip distribution, mode split and assignment steps of the UTFP [see Boyce et al (1)]. This type of model is not new; its adoption, however, in transportation planning practice is very slow. Transportation professionals seem to experience difficulty in understanding the solution procedure. In addition, research codes do not provide relief because they lack detailed documentation and require computer programming expertise to be adapted to professional practice. Moreover, software developers have ignored the issue simply because there has not been sufficient demand, at least until recently.

Several algorithms for solving the combined origin-destination, mode choice and user equilibrium traffic assignment model exist and their properties are well documented [see Florian (2) and Patriksson (3)]. Among those algorithms, the Frank-Wolfe linear approximation algorithm, and its variant the Evans partial linearization algorithm, have been applied to large scale urban networks. The algorithm implemented here is the one proposed by Evans (4); its advantages as compared to the Frank-Wolfe algorithm, especially for large scale applications, have been reported elsewhere [see Boyce et al (1), Florian (2), Frank (5), LeBlanc and Farhangian (6), Boyce et al (7) and Chu (8)]. In a recent study by Boyce et al (1), the Evans algorithm for the combined distribution, mode split and traffic assignment model was compared with various heuristics used in practice and provided superior results as defined by its more rapid convergence to the true equilibrium solution.

In this paper we consider the solution of a combined model of trip distribution, mode choice and user-equilibrium traffic assignment using the Evans algorithm. The model was solved and calibrated using 1980 data by Boyce et al (9). The implementation of the Evans algorithm in EMME/2 builds on a previous effort by Metaxatos et al (10). It is made possible by the capability of the EMME/2 macro language to utilize various modules for mathematical and network operations both sequentially and iteratively. The macro is subsequently tested with the 1990 Chicago sketch planning network data; the results are very good. Therefore, the macro now gives practitioners the option to introduce “feedback” in the UTFP in a rigorous way.

The presentation begins with the introduction of the cost functions used and continues with the formulation of the combined origin-destination, mode and route choice model and its optimality conditions. Then the Evans solution algorithm and its implementation in EMME/2 are documented in some detail. An evaluation of the results and a comparison with the sequential procedure is then discussed followed by some concluding remarks.

Cost Functions

The link volume-delay function used is the BPR function

\[
c_l(v_l) = c_l^0(1 + 0.15\left(\frac{v_l}{k_l}\right)^4)
\]  

(1)
where,
\( c_l(v_l) \) – travel time on link \( l \)
\( c_l^0 \) – the free-flow travel time on link \( l \)
\( v_l \) – auto flow on link \( l \)
\( k_l \) – capacity on link \( l \) determined by the level of service

The operating cost of automobiles consists of a variable part for fuel consumption that depends on the travel time of the vehicle as well as the distance traveled and a constant part representing the cost of starting the vehicle and parking. The variable part is defined as

\[
k_l = r(k_1 d_l + k_2 c_l) = r(k_1 d_l + k_2 c_l^0(1 + 0.15 v_l k_l^4))
\]

where,
\( k_l \) – auto operating cost on link \( l \)
\( r \) – in cents/gallon
\( k_1 \) – in gallons/mile
\( k_2 \) – in gallons/minute
\( d_l \) – link length in miles

Operating costs are expressed in terms of the average cost per vehicle, whereas times are expressed on a per person basis.

**Model Formulation**

Assuming that there is no interaction between the auto and transit modes and that the transit in- and out-of-vehicle travel times and travel costs used in the generalized cost are fixed, the equivalent optimization problem may now be written as follows:

Minimize with respect to \( f_r, P_{ijm} \):

\[
Z = \frac{R}{N} \sum_i \int_0^v c_i(x)dx + \sum_i \sum_j P_{ijt} c_{ijt} + \frac{1}{N} \sum_i \int_0^v k_l(x)dx + \sum_i \sum_j P_{ijt} k_{ijt} + \gamma_3 \sum_{i \neq j} (\sum_j P_{ijh} w_{ijh} + \sum_i \sum_j P_{ijt} w_{ijt}) + \gamma_4 \sum_i \sum_j P_{ijt} + \sum_i P_{ih} C_{ih} + \frac{1}{\mu} \sum_i \sum_j \sum_m P_{ijm} \ln \frac{P_{ijm}}{P_i P_j}
\]

subject to:

\[
\sum_{r \in R_{ij}} f_r = P_{ijh} N/R + T_{ij}
\]

\[
\sum_j \sum_m P_{ijm} = \bar{P}_i
\]

\[
\sum_i \sum_m P_{ijm} = \bar{P}_j
\]

\[
f_r \geq 0
\]

\[
v_l = \sum_i \sum_j \sum_{r \in R_{ij}} f_r \delta_r^l
\]
where,
\( c_l(v_l) \) – travel time on link \( l \)
\( k_l(v_l) \) – automobile operating cost on link \( l \)
\( R \) – auto occupancy (persons/vehicle)
\( N \) – total number of trips/hour
\( R_{ij} \) – set of highway routes between zones \( i \) and \( j \)
\( T_{ij} \) – number of truck trips/hour in automobile equivalent units
\( f_r \) – total vehicle flow on route \( r \) in automobile equivalent units/hour
\( \delta_l^r \) – equals 1 if link \( l \) belongs to route \( r \), and 0 otherwise
\( P_{ijh} \) – proportion of person trips between zones \( i \) and \( j \) by automobile
\( P_{ijt} \) – proportion of person trips between zones \( i \) and \( j \) by transit
\( w_{ijh} \) – out-of-vehicle travel time between zones \( i \) and \( j \) by automobile
\( c_{ijt} \) – in-vehicle travel time between zones \( i \) and \( j \) by transit
\( k_{ijt} \) – transit fare between zones \( i \) and \( j \)
\( w_{ijt} \) – out-of-vehicle travel time between zones \( i \) and \( j \) by transit
\( \gamma_1 \) – coefficient for in-vehicle travel time in generalized cost units per minute
\( \gamma_2 \) – coefficient for auto operating cost or transit fare in generalized cost units per cent
\( \gamma_3 \) – coefficient for out-of-vehicle travel time in generalized cost units per minute
\( \gamma_4 \) – coefficient for transit bias, a component of transit generalized cost
\( C_{iih} \) – the average generalized cost for intrazonal auto travel in zone \( i \).

Objective function (3) refers to the average cumulative travel cost, an artificial cost with no direct interpretation. The cost perceived by the users of the system is a combination of several factors such as in-vehicle travel time, monetary cost and out-of-vehicle travel time. We may then define a generalized cost which is the weighted sum of the different times and costs associated with each mode. It can be proved that the auto generalized cost from zone \( i \) to zone \( j \) is
\[
C_{ijh} = \gamma_1 \sum_l c_l(v_l) \delta_l^r + \frac{\gamma_2}{R} \sum_l k_l(v_l) \delta_l^r + \gamma_3 w_{ijh} \tag{9}
\]
whereas that for transit (which is fixed) is
\[
C_{ijt} \equiv \gamma_1 c_{ijt} + \gamma_2 k_{ijt} + \gamma_3 w_{ijt} + \gamma_4 \tag{10}
\]
The entropy term in (3) reflects dispersion of origin-destination and mode choices which result in non-cost-minimizing travel patterns similar to the observed patterns. Constraints (4)-(6) are conservation of flow constraints in terms or routes, origin and destination flows respectively. Constraints (7) are non-negativity constraints for the auto and transit flow proportions \( P_{ijmr} \).

**Optimality Conditions**

It can be proved that the optimality conditions for the above stated minimization problem correspond to Wardrop’s first principle for the route flows and that the trip proportions have the gravity model form. Thus for auto and transit the trip proportions are
\[
P_{ijm} = A_i P_i B_j P_j \exp(-\mu C_{ijm}) \tag{11}
\]
\[ A_i = \frac{1}{\sum_j \sum_m B_j \bar{P}_j \exp(-\mu C_{ijm})} \] (12)

\[ B_j = \frac{1}{\sum_i \sum_m A_i \bar{P}_i \exp(-\mu C_{ijm})} \] (13)

where \( m \) stands for \( h \) (auto) or \( t \) (transit) and \( C_{ijm} \) is given by (9) and (10).

**Solution Procedure**

The solution algorithm used for the combined origin-destination, mode and route choice model is the Evans algorithm. The algorithmic steps can be summarized as follows:

**Step 0:** Initialization: find initial trip proportions \( P_{ijm} \) from (11)-(13), and link flows \( v_l \) using an all-or-nothing assignment based on zero flow link costs.

**Step 1:** Update link costs based on the new flows \( v_l \).

**Step 2:** Find new minimal-cost routes based on costs from Step 1.

**Step 3:** Find the feasible descent direction by:

1. computing travel demands \( Q_{ijm} \) using the minimal-cost routes for auto, and solving for \( A_i \) and \( B_j \);
   \[ Q_{ijm} = A_i \bar{P}_i B_j \bar{P}_j \exp(-\mu C_{ijm}), \] (14)

2. computing link flows \( z_l \) by assigning travel demands \( (Q_{ijm}) \) to the minimal-cost routes.

**Step 4:** Conduct line search: find an optimal step size \( \lambda \) such that if \( x \) represents the current solution \( P_{ijm} \) and \( v_l \) and \( y \) represents the subproblem solution \( (Q_{ijm} \text{ and } z_l) \), then \( x' = x + \lambda (y - x) \) minimizes the objective function.

**Step 5:** Update trip proportions \( (P_{ijm}) \) and link flows \( (v_l) \) using the step size \( \lambda \) such that \( P'_{ijm} = (1 - \lambda) P_{ijm} + \lambda Q_{ijm} \) and \( v'_l = (1 - \lambda) v_l + \lambda z_l \). The costs based on the updated link flows are then used to find a new subproblem solution at step 2.

**Step 6:** Test convergence: Steps 2 to 5 are repeated until a convergence criterion is satisfied. The criterion used is that the current Relative Gap (see relevant section for a precise definition) should be lower than 0.01 of the current objective function value.

The implementation of the Evans algorithm in EMME/2 follows.

**Step 0: Initialization**

In this step we compute the initial solution for trip proportions and link flows. We also compute other parameters needed in the next steps. Therefore, we initialize:

1. an initial solution for demand, as \( m^* g_{ij0} = 1 \);
2. an initial solution for link flows, as \( u_{l1} = 0 \);
3. the start-up cost for the auto mode, as \( ms \text{“fixcst”} = 35 \text{ cents} \);

4. the origin-destination-specific access and egress auto costs, as \( mf \text{“accegr”} = mo \text{“cacces”} + md \text{“cegres”} \);

5. the mode specific generalized-cost coefficients \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \), as \( ms \text{“gam1h”}, ms \text{“gam2h”}, ms \text{“gam3h”} \) for auto, and \( ms \text{“gam1t”}, ms \text{“gam2t”}, ms \text{“gam3t”}, ms \text{“gam4t”} \) for transit;

6. the occupancy factor, as \( ms \text{“occ”} \);

7. the fuel consumption parameters for distance \( rk_1 \) and travel time \( rk_2 \), as \( ms \text{“rk1”} \) and \( ms \text{“rk2”} \), respectively. Because the auto operating cost is also a function of the link length, the latter is computed as

\[
ul3 = \frac{ms \text{“gam2h”}}{ms \text{“occ”}} \times ms \text{“rk1”} \times \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}
\]

(15)

(where \((x_i, y_i)\) and \((x_j, y_j)\) the coordinates of the link end-points), and added in the BPR function in the link function file. This was necessary in our case because all attributes available in the link file were used to store other link information. Care has to be taken so that the \((x,y)\) coordinates used to compute the link lengths give the true distances; otherwise a correction factor needs to be considered;

8. the deterrence parameter \( \mu \) for the simultaneous choice of origin-destination, mode and route, as \( ms \text{“mu”} \);

9. the auto intra-zonal travel time, cost, generalized cost, and distance for different zone sizes. Roughly speaking, the 1990 sketch planning zone system for the Chicago region consists of 9 and 36 square mile zones. The cases of external zones and the CBD are considered separately. Assume that the intra-zonal travel time and cost are initialized as \( mo \text{“intt”} \) and \( mo \text{“intc”} \), respectively. Then the auto generalized cost is computed as

\[
mo \text{“zcost”} = (ms \text{“gam1h”} \times mo \text{“intt”}) + (ms \text{“gam2h”} \times mo \text{“intc”})
\]

(16)

while the auto intra-zonal distance is computed as

\[
mo \text{“adistx”} = \frac{mo \text{“intc”} - (ms \text{“rk2”} \times mo \text{“inttx”})}{ms \text{“rk1”}}
\]

(17)

10. the trip proportions for the observed combined auto and transit demand, as \( mf \text{“propat”} \);

11. the productions and attractions for the combined auto and transit proportions, as \( mo \text{“prodat”}, md \text{“attrat”} \), respectively;

12. the destination-specific auto parking and start-up cost, as

\[
md \text{“cprkvh”} = \frac{md \text{“cpark”} + ms \text{“fixcst”}}{ms \text{“occ”}}
\]

(18)
13. the fixed origin-destination specific out-of-vehicle auto travel time, as
\[
mf^{"cijh"} = (ms^{"gam2h"} \times md^{"cprkvh"}) + (ms^{"gam3h"} \times mf^{"accegr"})
\] (19)

14. the origin-destination-specific fixed generalized cost for transit, as
\[
mf^{"cijt"} = (ms^{"gam1t"} \times mf^{"trivt"}) + (ms^{"gam2t"} \times mf^{"trfare"}) \\
+ (ms^{"gam3t"} \times mf^{"trovt"}) + ms^{"gamma4"}
\] (20)

where, \(mf^{"trivt"}\) the in-vehicle transit travel time, \(mf^{"trfare"}\) the transit fare, and \(mf^{"trovt"}\) the out-of-vehicle transit travel time. An exponential function of \(mf^{"cijt"}\) is also computed as
\[
mf^{"ecijt"} = \exp (0 - ms^{"mu"} \times mf^{"cijt"})
\] (21)

15. a zero demand matrix, as \(ms^{"zero"}\), which is used in the computation of the optimal step size and the objective function.

**Steps 1 and 2: Update Link Costs and Find New Minimal-Cost Routes**

The initial demand \(ms^{"gij0"}\) is assigned to the network and the inter-zonal travel generalized costs (skims) are saved as \(mf^{"cijhk"}\).

**Step 3: Find Feasible Descent Direction**

This step of the Evans algorithm involves the computation of the descent direction for the auto and transit trip proportions (demand term) and the link flows (network term). The descent direction for the demand term can be determined once the subproblem solution for the auto and transit proportions is computed. The latter is computed as follows:

1. The skims computed in Step 2 are added to the intra-zonal generalized travel costs computed in the initialization step because the intra-zonal auto demand is also assigned to the network. This involves filling the diagonal of \(mf^{"cijhk"}\). The full matrix \(mf^{"fcijhk"}\) is computed as
\[
mf^{"fcijhk"} = mf^{"cijhk"} \times (p.ne.q) + \sum g mo_g^{"zcost"} \times (p.eq.q)
\] (22)

where \(mf^{"cijhk"}\), the skims and \(mo_g^{"zcost"}\) the intra-zonal auto costs for each zone group \(g\). The expression \((p.eq.q)\) is used to denote that the algebraic operations will involve only intra-zonal matrix elements, that is elements with the same origin and destination.

2. Next, the in- and out-of-vehicle auto generalized costs are added together and the familiar exponential factor of the gravity model is computed as
\[
mf^{"ecijhk"} = \exp(0 - ms^{"mu"} \times (mf^{"fcijhk"} + mf^{"cijh"}))
\] (23)
3. The composite cost for auto and transit is then computed as

\[ mf^{\text{cc}} = mo^{\text{prodat}} \times md^{\text{attrat}} \times (mf^{\text{ecijh}} + mf^{\text{ecijt}}) \] (24)

4. A gravity model is solved to determine the subproblem auto and transit proportions. This is done by balancing the matrix \( mf^{\text{cc}} \) to the productions and attractions \( mo^{\text{prodat}} \) and \( md^{\text{attrat}} \), respectively. The balancing coefficients are saved as \( mo^{\text{ai}} \), \( md^{\text{bj}} \) and the subproblem auto trip proportions are computed as

\[ mf^{\text{wijh}} = mo^{\text{prodat}} \times md^{\text{attrat}} \times mf^{\text{ecijh}} \times mo^{\text{ai}} \times md^{\text{bj}} \] (25)

Similarly, the subproblem transit trip proportions are computed as

\[ mf^{\text{wijt}} = mo^{\text{prodat}} \times md^{\text{attrat}} \times mf^{\text{ecijt}} \times mo^{\text{ai}} \times md^{\text{bj}} \] (26)

5. Before the auto demand \( mf^{\text{wijh}} \) is assigned to obtain the subproblem link flows solution, it is increased by the number of truck vehicles. Thus the increased demand \( mf^{\text{inwijh}} = (mf^{\text{wijh}} \times ms^{\text{atveh}}) + mf^{\text{truck}} \) is assigned, where \( ms^{\text{atveh}} \) the vehicles equivalent auto and transit demand, and \( mf^{\text{truck}} \) the truck vehicles. The trucks are assigned using the additional demand option of module 5.11.

The subproblem link flows result from the last assignment of the increased auto demand \( mf^{\text{inwijh}} \). The descent direction for the link flows can then be determined. Finally, in iteration 1, \( mf^{\text{wijh}} \), \( mf^{\text{wijt}} \) and \( \text{volau} \) are designated to be the current solution \( mf^{\text{pijhk}} \), \( mf^{\text{pijtk}} \) and \( ul1 \), respectively. Therefore the direction of descent is always \( mf^{\text{wijh}} - mf^{\text{pijh}} \) for auto and \( mf^{\text{wijt}} - mf^{\text{pijtk}} \) for transit trip proportions, and \( (ul1 - \text{volau}) \) for the link flows. From iteration 2 and thereafter, a line search is conducted to average the solutions for successive iterations.

**Step 4: Conduct Line Search**

In this step we find the optimal \( \lambda \) so that a linear combination of the previous (main) problem and current (subproblem) solutions for auto and transit demand and link flows minimize the objective function (3). Therefore, we seek to

\[
\min_{0 \leq \lambda \leq 1} Z(\lambda) = \gamma_1 \left( R \sum_i \int_0^{v_i+\lambda(z_i-n_i)} c_i(x)dx + \sum_i \sum_j (P_{ijt} + \lambda(Q_{ijt} - P_{ijt}))c_{ijt} \right) + \\
\gamma_2 \left( \frac{1}{N} \sum_l \int_0^{v_l+\lambda(z_l-v_l)} k_l(x)dx + \sum_i \sum_j (P_{ijt} + \lambda(Q_{ijt} - P_{ijt}))k_{ijt} \right) + \\
\gamma_3 \left( \sum_{i \neq j} \sum_j (P_{ijh} + \lambda(Q_{ijh} - P_{ijh}))w_{ijh} + \sum_i \sum_j (P_{ijt} + \lambda(Q_{ijt} - P_{ijt}))w_{ijt} \right) + \\
\gamma_4 \sum_i \sum_j P_{ijt} + \lambda(Q_{ijt} - P_{ijt}) + \sum_i (P_{ijh} + \lambda(Q_{ijh} - P_{ih}))C_{ijh} + \\
\frac{1}{\mu} \sum_i \sum_j \sum_m (P_{ijm} + \lambda(Q_{ijm} - P_{ijm})) \ln \frac{P_{ijm} + \lambda(Q_{ijm} - P_{ijm})}{P_i P_j} \right) (27)
\]
This is done by setting the derivative of (27) to zero; that is,

\[
\frac{\partial Z(\lambda)}{\partial \lambda} = \gamma_1 \left\{ \frac{R}{N} \sum_l c_l (v_l + \lambda (z_l - v_l))(z_l - v_l) + \sum_i \sum_j (Q_{ijt} - P_{ijt}) c_{ijt} \right\} + \\
\gamma_2 \left\{ \frac{1}{N} \sum_l k_l (v_l + \lambda (z_l - v_l))(z_l - v_l) + \sum_i \sum_j (Q_{ijt} - P_{ijt}) k_{ijt} \right\} + \\
\gamma_3 \left[ \sum_{i \neq j} \sum_j (Q_{ijh} - P_{ijh}) w_{ijh} + \sum_i \sum_j (Q_{ijt} - P_{ijt}) w_{ijt} \right] + \sum_i \sum_j (Q_{ih} - P_{ih}) C_{ih} + \\
\frac{1}{\mu} \sum_i \sum_j \sum_m \ln \frac{P_{ijm} + \lambda (Q_{ijm} - P_{ijm})}{(Q_{ijm} - P_{ijm})} = 0 (28)
\]

The solution of (28) is found by conducting a one-dimensional (one \( \lambda \)) search on the \([0,1]\) interval using an (iterative) root-finding method. Here, the bisection method is used. The method evaluates (28) for \( \lambda = 0, 0.5, 1 \). The value of (28) at \( \lambda = 0 \) is always negative due mainly to the fact that the flows on shortest routes (subproblem link flows) are lower cost than the flows in the network (main problem link flows) since the former result from an all-or-nothing assignment of the subproblem demand (gravity model). If the value of (28) at \( \lambda = 0.5 \) is negative then in the next bisection iteration the one-dimensional search is limited to the interval \([0,0.5]\); if it is positive the bracketing interval is \([0.5,1]\). The bisection iterations continue and the search interval gets smaller and smaller until \( \lambda \) is close enough to the actual root. This \( \lambda \) then is designated as the optimal \( \lambda \) for that particular Evans iteration.

In the EMME/2 implementation of the algorithm the computations are done separately for the network term (the terms in (28) related to link flows) and the demand term (the rest of the terms in (28). As explained in greater detail in a previous paper (see Metaxatos et al (10)), the computation of the network term involves performing an assignment of a zero demand in a dummy scenario and adding up the resulting link travel costs. The network term is then added to the demand term computed in the working scenario using the matrix calculator. Once (28) has been computed a test for convergence of the bisection iteration is performed to find how close we are in finding the optimal \( \lambda \). Here, the expression evaluated is

\[
1 \times \left\{ \frac{\text{abs(ms”mid”} - \text{ms”mid-1”})}{\text{abs(ms”mid”})} - \text{ms”lamacc”} \leq 10^{-6} \right\} (29)
\]

where,

ms“mid” – the midpoint of the search interval at the current bisection iteration

ms“mid-1” – the midpoint of the search interval at the previous bisection iteration

ms“lamacc” – the required accuracy in finding the optimal \( \lambda \); here, \( 10^{-3} \)

The evaluation criterion (29) is a boolean expression, that is, it can take two values; 1 (true) or 0 (false). If the value is 1, then the optimal \( \lambda \) has been found with some reasonable accuracy (\( 10^{-3} \) here) and the bisection iteration terminates. If the value is 0, the search for the optimal \( \lambda \) continues in the next bisection iteration.
Step 5: Update Solution

The tasks in this step are straightforward. The auto demand from the previous Evans iteration is saved in mf“phk-1”. The current solution for the auto demand is then saved in mf“pijhk” as

$$mf \text{“pijhk”} = mf \text{“pijhk”} + ms \text{“mid”} \times (mf \text{“wijh”} - mf \text{“pijhk”})$$ (30)

where ms“mid” is the scalar that stores the current optimal $\lambda$. Similarly, for the transit demand the solution from the previous Evans iteration is saved in mf“ptk-1” and the current solution is computed in mf“pijtk” as

$$mf \text{“pijtk”} = mf \text{“pijtk”} + ms \text{“mid”} \times (mf \text{“wijt”} - mf \text{“pijtk”})$$ (31)

Finally, the update of the link flows solution is performed by saving the link flows solution from the previous Evans iteration in $ul2$ and computing the current solution in $ul1$ as

$$ul1 + %ms \text{“mid”}\% \times (volau - ul1)$$ (32)

where volau the EMME/2 link attribute that stores the current subproblem link flows solution.

Step 6: Test Convergence

A number of alternative convergence criteria have been computed. For the auto and transit demand and the link flows the maximum over all terms (origins, destinations or links) of the absolute deviations between the current solution and the solution from the previous iteration is computed. Thus, for the auto demand the computation is

$$ms \text{“phdif”} = abs(mf \text{“pijhk”} - mf \text{“phk-1”})$$ (33)

and for the transit demand

$$ms \text{“ptdif”} = abs(mf \text{“pijtk”} - mf \text{“ptk-1”})$$ (34)

while for the link flows the computation is

$$ms \text{“vdif”} = abs(ul1 - ul2)$$ (35)

The following convergence criteria are strongly recommended. First, the objective function in (3) at the current iteration $OF^k$ is evaluated. The objective function consists of the network term (terms with link flow variables) and the demand term the computation of which is done separately in the network (module 2.41) and matrix (module 3.21) calculators, respectively. The computation of the network term involves evaluation of an integral. This is accomplished in EMME/2 by defining a new volume-delay function which corresponds to the integral of the network term. Using this volume-delay function in the dummy scenario an all-or-nothing assignment of a zero demand is performed and the resulting generalized travel costs saved in the EMME/2 attribute timau are added link by link and saved in a scalar.
The demand term of the objective function is then computed using the matrix calculator and added to the network term resulting in the current value of the objective function, $OF^k$.

Next the current value of $GAP^k$ is computed. At each iteration of the Evans algorithm, the subproblem solution provides a lower bound $LB^k$ for the objective function value $OF^k$. That is, the current GAP defines the distance from the current value of the objective function to the lower bound. In other words, $LB^k = OF^k + GAP^k$. The current GAP is simply the value of (28) corresponding to $\lambda = 0$ and is increasing from large negative values to zero (at equilibrium), but not monotonically. $LB^k$ is decreasing to zero (at equilibrium), also not monotonically. The Best Lower Bound is defined as

$$BLB^k = \min_k (LB^k)$$

that is, the lowest lower bound found up to Evans iteration $k$. The Relative GAP at iteration $k$, $RELGAP^k$, is now defined as

$$RELGAP^k = \left| \frac{GAP^k}{BLB^k} \right|$$

that is, the absolute value of the ratio of GAP to Best Lower Bound at iteration $k$. The absolute value eliminates the negative sign of GAP and a possible negative sign of BLB at early Evans iterations. Desirably, the combined model should be solved to a prespecified value of RELGAP such as 0.01. The number of iterations required increases with the level of congestion in the network. For our Chicago Region model, about 20 iterations are required to reach 0.01 using one all-or-nothing assignment per iteration.

Two other criteria which are also computed is the transit share, which is simply the sum over all origins and destinations of $mf^{pijtk}$, and the average auto generalized cost. The performance of all the above convergence criteria is illustrated in Figure 1.

**Evaluation of the Results and Comparisons with the Sequential Procedure**

The macro has been tested with the 1990 Chicago sketch planning network developed jointly by the Chicago Area Transportation Study (CATS) and the Northeastern Illinois Planning Commission (NIPC). The sketch planning zone system was based on the CATS 1640-zone regional system and consists of 387 zones, which represent aggregations of City of Chicago community areas, survey townships, survey quarter-townships, political townships, and buffer areas as large as 206 square miles. The sketch planning network is an aggregation of the CATS regional network and consists of 546 regular nodes (in addition to the 387 centroids) and 2950 one-way links, including artificial links connecting the centroids to the network.

In Figure 1 the convergence properties of the Evans and the Method of Successive Averages (MSA) algorithms are illustrated. The difference in the MSA algorithm (also known as method of predetermined step sizes) is in the computation of the step size. While the Evans algorithm moves along the current feasible descent direction with steps so that the objective function is minimized, the steps in the MSA algorithm are predetermined. In other words, the weights used to average successive iterations in the Evans algorithm are optimal in the sense that the objective function cannot get lower at the current iteration; in
the MSA algorithm, solution $k$, which is based on the result of iteration $k - 1$, is weighted by $1/k$ and the former solution is weighted by $(k - 1)/k$. The impact of choosing the best weights is illustrated in Figure 1. In one of the graphs the objective is monitored for the Evans and MSA algorithms and the traditional sequential procedure. Clearly, the Evans algorithm gives the lowest value of the objective function, while the sequential procedure by far the highest.

In Figure 2 another comparison between the combined origin-destination, mode and route choice and the traditional sequential procedure is illustrated. In the absence of observed link flow data we have solved the macro for 20 iterations and saved the obtained link flows in $ul3$ (horizontal axis). Then we estimated a trip distribution model based on free flow travel times and balanced to the production and attraction totals of the observed auto demand matrix. The estimated trips were then assigned to the network for 20 iterations and the link flows obtained were saved in $ul1$ (vertical axis). The plot shows that the link flows from the combined model are lower (better converged) than from a trip table based on free flow travel times. If the two methods were equivalent, the points would lie on the line shown in the figure. Since the points lie above the line, the link flows from the four-step procedure are higher which results from the longer trips based on free flow travel times.

**Conclusions**

The solution of a combined model of origin-destination, mode and route choice using a commercial software package provides an opportunity for some afterthoughts. Clearly, it is possible to solve a model routinely that rigorously introduces “feedback” in the traditional sequential procedure using modest computing resources. This is an important result for practitioners who are challenged with increased modeling requirements.

We recognize that the four-step models solved in practice are considerably more elaborate than the combined model presented here with regard to the disaggregation of the trip tables by mode and purpose. We believe, however, that the latter can be accomplished in a manner similar with this presentation. Meanwhile, we feel that the present implementation of a combined model in EMME/2 can meet some of the modeling requirements arising from modern urban transportation planning practice and motivate transportation professionals to use more sound planning methods. The quality of the results obtained to date seem to encourage the use of the macro in planning studies.

**REFERENCES**


FIGURE 1  Convergence of the Evans and the Method of Successive Averages algorithms.
FIGURE 1 (continued)
FIGURE 2  Link flows from the sequential procedure (y-axis) versus link flows from the combined model (x-axis).